SHENTON COLLEGE

2019 YEAR 12 MATHEMATICS: METHODS
Test 3 (Continuous Random Variables, Normal Distribution, Logarithms)

NAME:_____

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Calculator-Free **Formula sheet provided** **Working time: 25 minutes** **Marks: 41**

QUESTION 1

[5 marks – 2, 3]

Evaluate the following logarithms.

a) $\frac{\log_5 8}{\log_5 32}$	b) $2\log_6 3 - \log_6 54 + 2$

QUESTION 2

[10 marks – 2, 3, 2, 3]

a) If $\log_a 3 = x$ and $\log_a 5 = y$, express the following in terms of x and y.

i) $\log_a(3\sqrt{5})$	ii) $\log_a\left(\frac{9}{5a}\right)$

b) If $\log m = 7$ and $\log n = 4$, evaluate the following.

i) $\log(mn^3)$	ii) $\log\left(\frac{100\sqrt{m}}{n}\right)$

QUESTION 3

[8 marks – 3, 2, 3]

a) Solve the following equation, stating your answer in terms of **base ten logarithms**.

$$3^{7x-2} = 5^{x+1}$$

b) Solve the following equations, stating your answers in terms of **natural logarithms**.

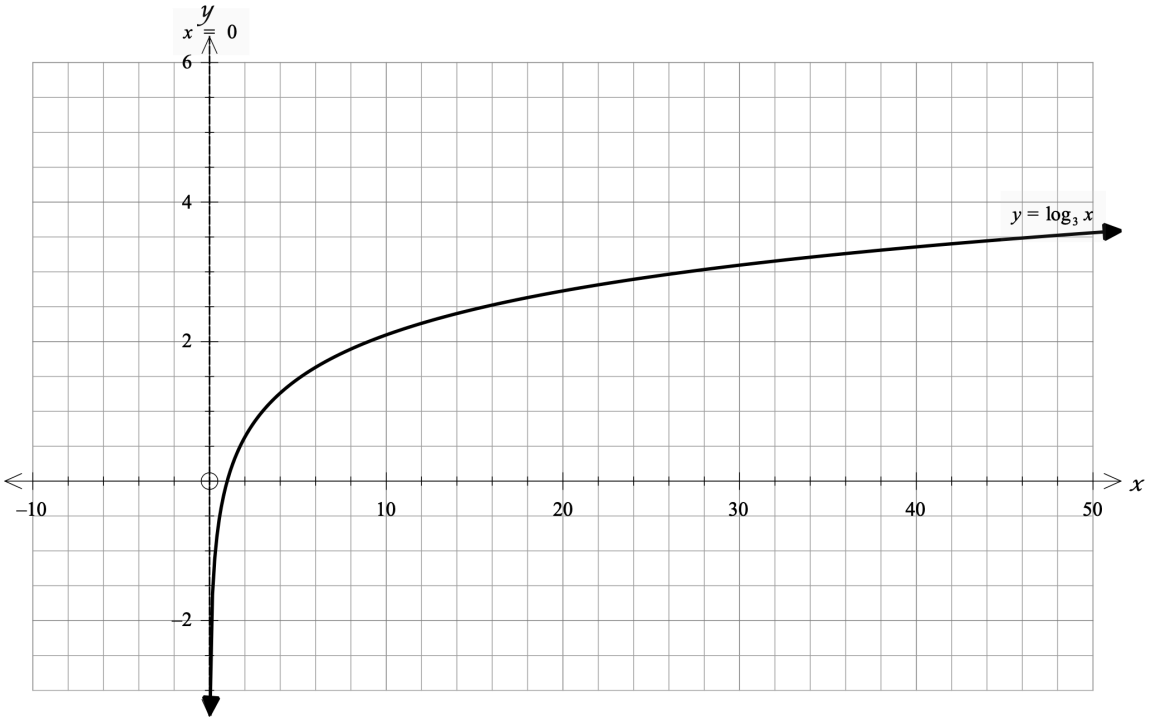
i) $e^{x+1} = 19$

ii) $2e^{2x} - 3e^{x} = 2$

QUESTION 4

[9 marks – 1, 2, 2, 2, 2]

The graph of $y = \log_3 x$ is shown below.



a) Use the graph above to solve for the approximate solution to $\log_3 x = 2.5$.

b) Use the graph above to approximate the solutions to $\log_3(x - 8) = 3.25$.

c)

i) If $y = \log_3 x$ is translated 27 units to the right and 2 units up, state its new equation.

ii) State the equation of the asymptote and the coordinates of the x-intercept of the new function.

iii) Add the sketch of the translated function onto the axes above, labelling its key features. Also label the coordinates of two other points.

QUESTION 5

[6 marks – 2, 1, 1, 1, 1]

A uniform continuous random variable X is defined over the interval $5 \leq x \leq 15$.

a) State its probability density function.

b) State the mean of X.

c) The variance of X is $\frac{280}{3}$. Write the definite integral that can be used to obtain this value.

d) The continuous random variable of Y is such that $Y = 3X + 2$

i) State the mean of Y

ii) State the variance of Y

QUESTION 6 Don't

[3 marks – 1, 2]

Use the 68%, 95%, 99.7% rule to calculate the following probabilities for $X \sim N(0,1)$.

a) $P(X \geq 3)$	b) $P(-2 < X < 1)$

End of Calculator Free Section